P P SAVANI UNIVERSITY

First Semester of B. Tech. Examination

May 2019

SESH1010 Elementary Mathematics for Engineers

25.05.2019, Saturday **Time: 12:30 p.m. To 3:00 p.m.** **Maximum Marks: 60**

Instructions:

1. The question paper comprises of two sections.
2. Section I and II must be attempted in separate answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

Q - 1 **Do as directed: (Any Five)** **[05]**

(i) Find $\lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}}$.

(ii) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ is convergent or divergent.

(iii) Write an example of a curve which is symmetric about x-axis.

(iv) State the general form of the series $\frac{-1}{1+1} + \frac{2}{8+1} - \frac{3}{27+1} + \frac{4}{64+1} - \cdots$

(v) Check whether the graph of the cartesian curve $x^2 + y^2 = 4$ is symmetric or not? If so, mention the axis/lines/points of its symmetry.

(vi) Find $\lim_{x\to\infty} \frac{3x^2-x-2}{5x^2+4x+1}$

(vii) State the relation between polar and cartesian coordinates.

Q - 2 (a) Show that the sequence $\{u_n\}$, where $u_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{(n-1)!}$; $n \geq 2$, is convergent. **[05]**

Q - 2 (b) State Lagrange's Mean Value Theorem and hence verify it for $f(x) = lx^2 + mx + n$ in [a,b] **[05]**

OR

Q - 2 (a) **[05]**

i) Expand e^x in power of $(x-1)$.

ii) Evaluate $\lim_{x\to 1}(\frac{x}{x-1} - \frac{1}{logx})$

Q - 2 (b) Trace the curve r = a (1+ cos θ). **[05]**

Q - 3 (a) Find the n^th^ derivative of $x^2 logx$ **[05]**

Q - 3 (b) Given the series $\sum_{n=1}^{\infty} \frac{1}{2^{n+1+(-1)^n}}$, show that **[05]**

i) Show that ratio test fails for this series

ii) Using root test, determine whether the series converges or diverges

OR

Q - 3 (a) Trace the curve $y^2(2a - x) = x^3$ with necessary steps. **[05]**

Q - 3 (b) Test the convergence of following series: **[05]**

i) $\sum_{n=1}^{\infty} \frac{n+1}{n^2} x^n$

ii) $\sum_{n=1}^{\infty} \frac{1}{n(logn)^2}$

Q - 4	**Attempt any one:**	
(i)	Obtain the points of discontinuity of a function f defined on $[0,1]$ as follows: $f(0) = 0, f(x) = \left(\frac{1}{2}\right) - x, if\ 0 < x < \frac{1}{2}, f\left(\frac{1}{2}\right) = \frac{1}{2}, f(x) = \left(\frac{2}{3}\right) - x, if \frac{1}{2} < x < 1,$ and $f(1) = 1.$	**[05]**
(ii)	Write the procedure for tracing of cartesian curves.	

SECTION – II

Q - 1	**Do as directed: (Any Five)**	**[05]**
(i)	Give an example of a homogeneous function.	
(ii)	Compute $\Gamma(10)$	
(iii)	Evaluate $\int_1^2 \int_3^5 x^2 y\ dydx$	
(iv)	Find $\frac{\partial f}{\partial z}$ at (1,4,-3) for $f(x, y, z) = x^2y^2z^2$.	
(v)	Give an example of an improper integral	
(vi)	Define Beta Function.	
(vii)	What is the domain of the function $z = \sqrt{y - x^2}$?	
Q - 2 (a)	Prove that : i) $\beta(m, n) = \beta(m, n + 1) + \beta(m + 1, n).$ ii) $\int_a^b (x - a)^{l-1}(b - x)^{m-1}dx = (b - a)^{l+m-1}\beta(l, m)$, where $(l, m > 0, a > b)$.	**[05]**
Q - 2 (b)	Evaluate $\iint e^{2x+3y}\, dA$, where the region of integration is a triangle bounded by the lines $x = 0, y = 0, x + y = 1.$	**[05]**
	OR	
Q - 2 (a)	Verify Euler's Theorem when $u=f\ (x,y) =\frac{x^2+y^2}{x+y}$	**[05]**
Q - 2 (b)	Evaluate: $\int\int \frac{xy}{\sqrt{1-y^2}} dxdy$ *over the first quadrant of the circle* $x^2 + y^2 = 1.$	**[05]**
Q - 3 (a)	Using the transformation $x + y = u, y = uv$, show that $$\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}}\ dy\, dx = \frac{e-1}{2}$$	**[05]**
Q - 3 (b)	If $x = rcos\theta$ and $y = rsin\theta$, show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$	**[05]**
	OR	
Q - 3 (a)	Find the volume of the cone of height h and base radius a.	**[05]**
Q - 3 (b)	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$	**[05]**
Q - 4	**Attempt any two:**	**[05]**
(i)	Evaluate $\int_0^{\pi/2} \sin^9\theta \cos^7\theta\ d\theta$.	
(ii)	Find the area enclosed within the curves $y = 2 - x$ and $y^2 = 2(2 - x)$.	
(iii)	Find f_u, f_v, f_w for $f(u, v, w)=\frac{u^2-v^2}{v^2+w^2}$.	

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